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series expressed by it, commences, *in all cases*, with a finite quantity, such that the terms of the series, by continual diminution, tend to zero.

The circumstances are very different with respect to $1^2 - 2^2g + 3^2g^2 - 4^2g^3 + \&c.$ As observed in the foot-note at p. 44, the commencement of convergency, in the limiting case, is at a term infinitely distant from the origin of the proposed series, and infinitely great. What analogy can there be between the *general* converging series—if it may be so called—of which this is a limiting case, and ordinary convergent series? And can it be affirmed, of any one of its cases, that the terms necessarily tend to zero? The answers to these questions will, I think, destroy all idea of analogy in such examples as those adduced above.

I have been compelled, in several parts of the present Paper, to dissent from certain doctrines and opinions promulgated by some very distinguished writers on analysis. In developing the principles and views here submitted to the Royal Irish Academy, I could not easily avoid a reference to these. I trust, however, that I have done so in no captious or uncandid spirit: I have only been anxious to arrive at truth in an inquiry of acknowledged perplexity, and of interest, perhaps, in the estimation of some, sufficient to justify the attempt. There are one or two points of analytical delicacy involved in this inquiry, which may perhaps be open to further discussion: if I have myself fallen into error in my treatment of these, I hope I shall be indulged with the same candour and consideration which I have endeavoured to exercise towards others.

Professor Mac Cullagh made a communication on the subject of Total Reflexion.

In the case of total reflexion the vibrations which take place in the rarer medium are in general elliptical, and when this medium is a crystal, the equations by which the ellipse of vibration is determined are very complicated. The projection

of this ellipse upon the plane of incidence may, however, be easily found by the remark in p. 102 of the present volume ; the projecting cylinder is therefore known, and as the ellipse of vibration is a section of this cylinder, the question of determining the ellipse is reduced to that of determining its plane. For this purpose Mr. Mac Cullagh gave the following rule. Having constructed the ellipsoid of *indices* (that whose axes are parallel to the axes of elasticity, and inversely proportional to the three principal velocities of propagation in the crystal) let its two planes of circular section intersect the aforesaid cylinder. The curves of intersection will be ellipses, which shall be supposed to have a common centre O in the axis of the cylinder. Let OP, OP' be the greater semiaxes of these ellipses, and OQ, OQ' the less semiaxes ; the lengths of the two former being denoted by p, p' , and the lengths of the two latter by q, q' . Join the extremities P, P' of the greater semiaxes, and the extremities Q, Q' of the less semiaxes ; and divide each of the right lines PP', QQ', in the ratio of $\sqrt{p^2 - q^2}$ to $\sqrt{p'^2 - q'^2}$. Then a plane drawn through the centre O and the two points of division will be the plane of vibration. In the application of this rule some precautions are to be observed, but they need not here be insisted on.

The foregoing rule was deduced (in the year 1843) from the general equations by a peculiar use of imaginary quantities, after the author had several times tried in vain to obtain a geometrical interpretation of those equations by considerations of a more obvious and ordinary kind. This use of imaginaries is founded on a remarkable theorem relative to the ellipse, by which it appears, that the *plane* of an ellipse and its *species* (that is, the directions and the ratio of its axes) may be expressed by two imaginary constants, just as the direction of a right line in space is expressed by two real constants. By means of this theorem—which it is unnecessary to repeat, as it has been published in the University Calendar (Examination Papers of the year 1842, p. lxxxiv.)—we may find such

properties of elliptical vibrations as are analogous to those of rectilinear vibrations; and it was in this way that the above rule was discovered. It is analogous (though it scarcely appears so at first sight) to the rule by which, in the theory of Fresnel, the direction of rectilinear vibrations is determined, when the plane of the wave is given.

The Rev. Charles Graves read the first part of a paper on Algebraic Triplets.

The object which he proposes to himself is to frame, for the geometry of three dimensions, a theory strictly analogous to that by which Mr. Warren has succeeded in representing the combined lengths and directions of right lines in a plane. In carrying out this design Mr. Graves has necessarily been led to the consideration of new imaginaries.

For the sake of clearness it will be desirable to take, in the first instance, a brief survey of the fundamental properties of algebraic couplets, depending, as they do, upon the nature of the symbol $\sqrt{-1}$. The correspondence between received notions and the views now put forward will thus be made more apparent.

If we take the binomial or couplet $x + \sqrt{-1} \cdot y$, in which x and y are real quantities, and multiply it by a similar couplet $x_1 + \sqrt{-1} \cdot y_1$, the product will likewise be a binomial of the same kind, $x_2 + \sqrt{-1} \cdot y_2$; and between the constituents of the three couplets there exists the relation

$$(x^2 + y^2) (x_1^2 + y_1^2) = x_2^2 + y_2^2. \quad (a)$$

But couplets may be more readily compared after undergoing a simple transformation. Such an expression as $x + \sqrt{-1} \cdot y$ may be reduced to the form $re^{\sqrt{-1} \cdot \theta}$ by making $r = \sqrt{x^2 + y^2}$, and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. Hence it appears that if we agree to call r the *modulus* and θ the *amplitude* of a couplet, the following theorems will be true: